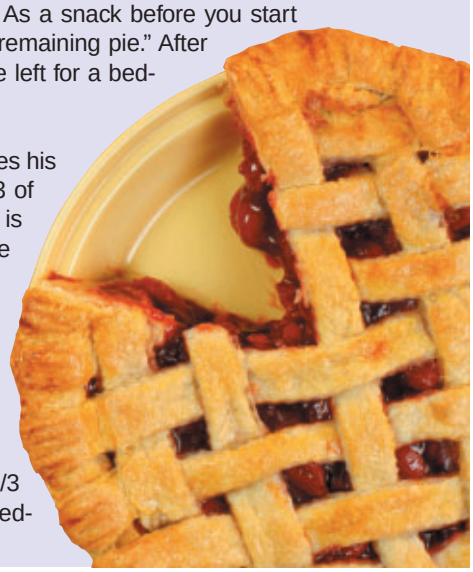
Fractionberry Pie

PROBLEM

Mrs. Pauwels poses the following problem to her students:

> When you arrive home after a long day at school, you find 6/8 of a fractionberry pie sitting on the counter with a note. The note says, "Welcome home! As a snack before you start your mathematics homework, you may eat 2/3 of the remaining pie." After you eat your snack, how much fractionberry pie will be left for a bedtime treat?



After working on the problem for a few minutes, Bob shares his solution with his partner, Kim. Bob says, "I get to eat 2/3 of the pie on the counter when I get home. I know that 2/3 is the same as 6/9. 6/8 and 6/9 are very close to the same amount, so after my snack I don't think there will be any pie left. My answer is 0—no pie left for a bedtime treat."

- Kim is not sure that Bob's thinking is correct. His answer to the problem doesn't match hers. Is Bob's solution reasonable? Explain.
- What do you think Kim's solution is? Explain.
- What if 6/7 of a pie was on the counter? After eating 2/3 of the remaining pie, how much pie would be left for a bedtime treat? Explain.

Mary Swarthout

The goal of the "Problem Solvers" department is to foster improved communication among teachers by posing one problem each month for K–6 teachers to try with their students. Every teacher can become an author: pose the problem, reflect on your students' work, analyze the classroom dialogue, and submit the resulting insights to this department. Every teacher can help us all better understand children's capabilities and thinking about mathematics with their contributions to the journal. Remember that even student misconceptions provide valuable information.

Edited by **Mary Swarthout,** *MB-Swarthout@wiu.edu,* **Robert Mann,** *RR-Mann@wiu.edu, and* **Kim Hartweg,** *KK-Hartweg@wiu.edu, Department of Mathematics, Western Illinois University, Macomb, IL 61455-1390*

Classroom Setup

Begin by spending time discussing this problem with your students. Although the students may need help to understand the problem and the specified constraints, do your best to avoid giving too much guidance. Encourage your students to use words, manipulatives, pictures, tables, or other methods to experiment, organize, and explain their thinking about the problem. Collect student work, make notes about interactions that took place, and document the variety of student approaches that you observed and the kinds of questions that you and your students asked during the problem-solving process. Feel free to adapt the problem to fit the level and experience of your students. This may mean having students work in pairs rather than individually, supplying students with manipulatives that illustrate fractions, substituting different fractions in the problem, or changing the context of the problem from food to another option.

Allow the students time to discuss and share their solutions and strategies for solving the problem. Can they explain their thinking about Bob's solution? Can they describe how Kim might have solved the problem? The words that students use to explain their reasoning can illuminate what they understand about the concept of equivalent fractions. As you reflect on the problem and your experience interacting with the students, keep in mind the following questions:

- What difficulties did students have in understanding the problem?
- What strategies did students try?
- Were any student responses surprising to you?
- Did your students come up with any new challenges or extensions to the problem?

We are interested in how your students responded to the problem or your adaptation of the problem and how they explained or justified their reasoning. Please send us your thoughts and reflections. Include information about how you posed the problem and samples of students' work. Send your results with your name, grade level, and school by **1 December 2002** to Mary Swarthout, Mathematics Department, Western Illinois University, Macomb, IL 61455. Classrooms that respond will receive a special "Problem Solvers" certificate to acknowledge their effort. Selected submissions will be published in a subsequent issue of *Teaching Children Mathematics* and acknowledged by name, grade level, and school unless otherwise indicated.

Where's the Math?

The Fractionberry Pie problem places attention on the Communication Standard presented in *Principles and Standards for School Mathematics* (NCTM 2000). The hope is that students can "analyze and evaluate the mathematical thinking and strategies of others" and "use the language of mathematics to express mathematical ideas precisely" (p. 60). The problem also addresses the Content Standard, which states that students should "use models, benchmarks, and equivalent forms to judge the size of fractions" (p. 148). Gaining an understanding of fractions as quantities is a difficult task for students; the concept of equivalent fractions is important to students' comprehension. Many students may apply the rule for finding equivalent fractions—multiply or divide the numerator and denominator by the same amount—without understanding that equivalence is based on the use of the same unit whole. Six-ninths and six-eighths are close to the same amount only if the same whole is being compared, such as a whole fractionberry pie, sheet of paper, length of string, or set of counters. It is important to note that the unit of focus changes in multiplying and dividing fractions. In the posed problem, six-eighths is a name for an amount from the whole pie, but the two-thirds that we are interested in is two of three equal parts of the six-eighths, not of the original whole pie. Knowing the unit is essential to good fraction number sense, especially when applying the operations of addition, subtraction, multiplication, and division.

Reference

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000. ▲

(Solutions to a previous problem begin on the next page.)

Show Off Your Students!

Each month, the "Problem Solvers" department features a challenging mathematics problem for you to try with your students. Take notes as your students work on the problem, and share with us the insights that you gain from watching their efforts. The editors of the department compile teachers' reflections and their students results, and a short report about these efforts appears in a subsequent issue of *Teaching Children Mathematics.* Sharing your students' work in "Problem Solvers" is an exciting way to add to the journal, and it is fun to see your students' work in print! Details on how to contribute are included with each month's problem.