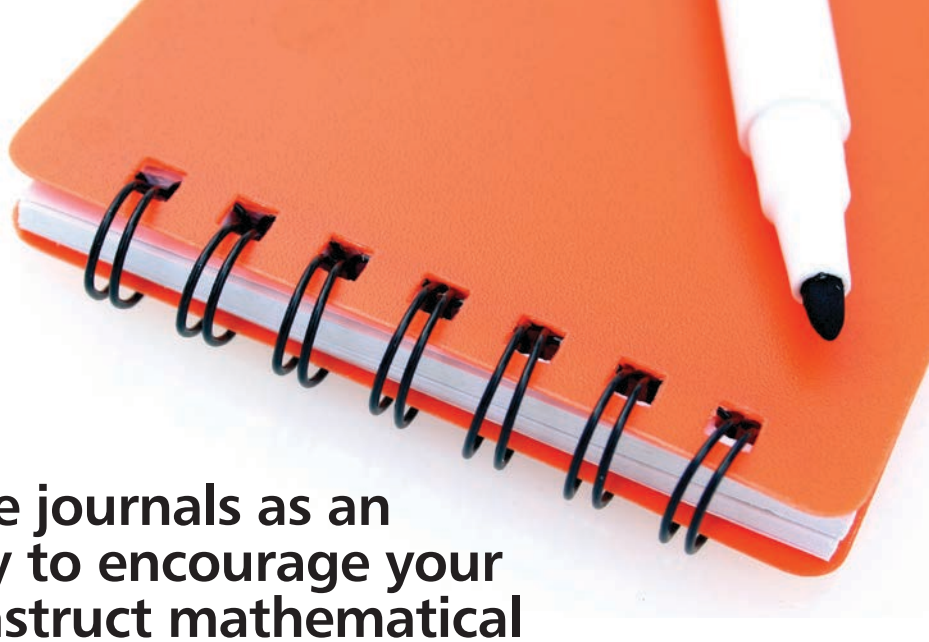
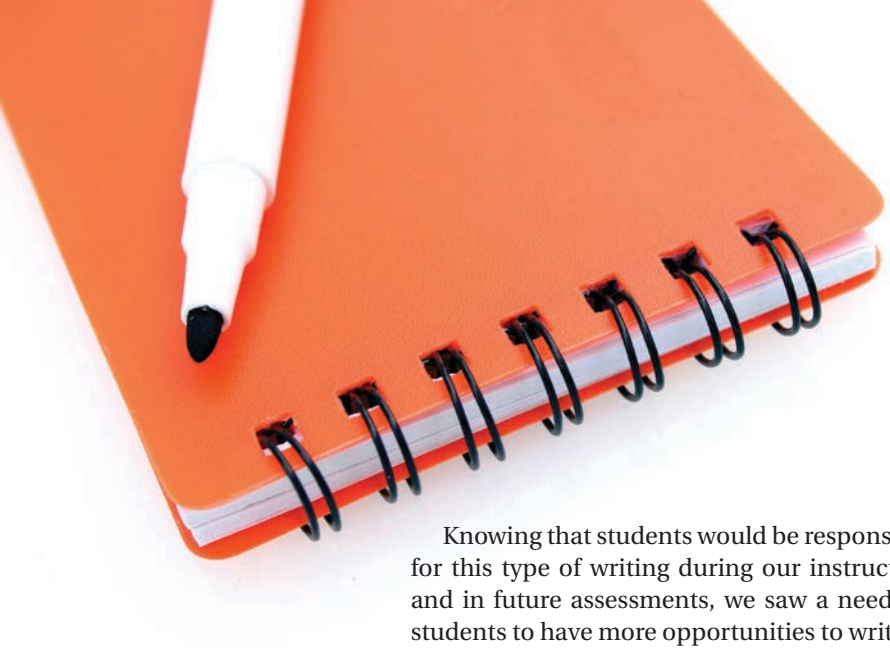
Moving Math in the Write Direction



Use debate journals as an innovative way to encourage your students to construct mathematical arguments as well as examine and critique others' mathematical thinking.

Shannon E. Bostiga, Michelle L. Cantin, Cristina V. Fontana, and Tutita M. Casa

Communication is a critical skill for students to possess in preparation for college and their future careers. One way for students to communicate their ideas in all content areas is through writing. The English language arts portion of the Common Core State Standards (CCSSI 2010a) specifically states that students need to understand perspectives other than their own and comprehend, critique, and share their thinking through writing. Similarly, the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010b) calls for students to convey their mathematical ideas and understandings of concepts to their peers and to their teachers. Using writing as a communication medium helps students develop not only their English language arts skills but also their mathematical competencies (McCormick 2010). When writing, students must articulate their thoughts and put them in print, a process that ultimately helps them evaluate themselves as learners (Bloem 2004). The process of writing emphasizes gathering, organizing, revising, and clarifying thoughts, all of which endeavor to fulfill the Common Core's Standards for Mathematical Practice (SMP), particularly the third one, which emphasizes fashioning valid arguments while considering and critiquing others' rationalizations (2010b). Writing also can serve as a platform for students to convey their reasoning in a clear, organized manner. CCSSM as well as accompanying assessments set forth such expectations for students. These related assessments require students to do more than simply select one response from multiple-choice questions. Rather, students are asked about their reasoning and are obligated to convey this in writing. (Visit http://www.parcconline.org and http://www.smarterbalanced.org for more information.)



Knowing that students would be responsible for this type of writing during our instruction and in future assessments, we saw a need for students to have more opportunities to write in math class. In particular, because—

> mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is (CCSSI 2010b, p. 6),

we decided to implement debate journals. In these journals, we presented students with prompts in which they were expected to explain their reasoning in writing in a format that compelled them to consider multiple sides of a debate. This way of presenting questions had students "construct viable arguments and critique the reasoning of others" (CCSSI 2010b, p. 6). These types of prompts required students to take a position based on different viewpoints and provide evidence supporting their claims.

The first three authors each worked with three small groups of five students: two groups of fifth graders and one group of third graders. As we worked with these students over time, we saw great improvements in our students' reasoning and written communication. We share our experiences and some considerations for other teachers to take into account as they implement math debate journals in their own classrooms to get their students "moving in the 'write' direction."

Write meaningful prompts

We found that a large component of starting the debate journal process is writing prompts that allow students to delve deeply into the mathematics (see **fig. 1**).

Common misconceptions

To ensure that students had opportunities to critique others' reasoning and construct their own arguments, we based prompts on common misconceptions. For example, many students have difficulties with adding fractions. A common yet incorrect approach is to add the numerators, then add the denominators without considering the role of the denominator and its relationship to the numerator. When presented with the Becky-and-Lisa prompt, our students were not permitted to just state that Becky is wrong because the denominators are not the same, so she cannot simply add the numerators. Instead, we seek to have students debate both perspectives by considering each response and using their knowledge to prove that their reasoning is correct. (We will later address the discussion portion of the intervention that allowed us to redirect any student who initially may have believed the presented misconception.) By creating prompts based on misconceptions, we could get students to focus in depth on the significance of the concept, such as the meaning and role of the denominator. Predicting students' misconceptions helped us understand what students might be thinking when they approach the prompt.

FIGURE 1

Incorporating journal prompts that focus on misconceptions and include both correct or both incorrect arguments forces students to reason for themselves instead of depending on patterns in the prompts.

- One argument is correct:
  Becky says that 1/2 + 1/3 is 2/5.
  Lisa says that 1/2 + 1/3 is 5/6.
  Do you agree with Becky or Lisa?
  Explain why.
- Both arguments are correct:
  Greg rounded 2.345 to 2.35.
  Sean rounded 2.345 to 2.3. Do you agree with Greg or Sean?
  Explain why.
- Both arguments are incorrect:
  In the number 6.534, Michael says the 3 has a value of 3/10. Brittany says it has a value of 3. Do you agree with Michael or Brittany?
  Explain why.

Consistent format

When writing prompts to enhance classroom instruction, considering their format is important. Using a debate format required students to view different perspectives presented in the prompts, which may or may not have coincided with their own thoughts. We wrote all our prompts, like the previous one that focuses on fractions, in a specific way. In general, the prompts read,

> Given the problem [or picture, etc.], student A says . . . , and student B says. . . . Who do you agree with and why?

Prompting students to look at both sides of an argument and consider different viewpoints addresses SMP 3, which calls for students to *justify their conclusions, communicate them to others, and respond to the arguments of others* (CCSSIb 2010, pp. 6–7). By doing so, students could take feedback that we provided and transfer it from one prompt to the next.

Changing the format along with the content material might have hindered writing growth among students. Nonetheless, as **figure 1** shows, we sought to further ensure that students reasoned in depth about the given concepts by incorporating entries in which not only one perspective was correct but also in which arguments were either both correct or both incorrect. This forced students to consider multiple viewpoints and truly rely on their reasoning instead of any given pattern in the prompts.

Content knowledge

Prompts must be deep enough for debate and reasoning but not so deep that students do not have the level of understanding necessary to write. Even with careful planning, at times students may not have as much content knowledge as they need for a specific prompt, such as the following:

> Jania simplified 18/54 to 6/9. Lewis simplified 18/54 to 2/3. Whose idea do you agree with and why?

Although students had been exposed to simplifying fractions, they were unable to digest what this problem was asking. Many could not make the jump from knowing how to simplify to knowing how *someone else* simplified. Consequently, students were quickly discouraged with the prompt and did not provide responses that were reflective of the work they had been producing. The lack of content knowledge got in the way of students' reasoning abilities. Their reasoning improved over time as we got to know our students better as well as the level of challenge that would be most appropriate for them.

Discuss the math before writing

We found that having students engage in a group discussion before writing was crucial because it tapped into and furthered their content knowledge and also supported them with brainstorming ideas and organizing their thinking. As Wenrick, Behrend, and Mohs (2013, p. 359) assert,

> initially, children communicate through oral language . . . [and] over time, children also learn to effectively communicate their ideas through written language.

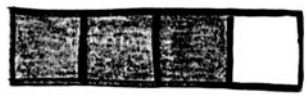
However, we felt that we had to be more systematic in helping students transfer their

FIGURE 2

In her response, Danielle demonstrated reasoning and elaboration.

Julia drew $\frac{3}{4}$ as

Ryan drew $\frac{3}{4}$ as



Do you agree with Julia's or Ryan's picture? Explain why.

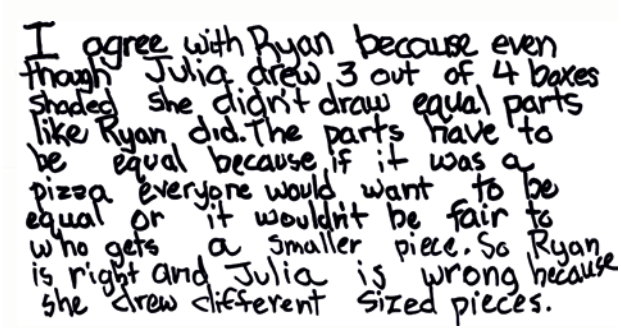


FIGURE 3

Students were presented with sample responses.

Question
Students in Ms. Bantom's class were presented with three digits: 6, 2, and 5. She asked students to make the smallest possible number using the three digits and decimals. Alex made the number 0.256. Dayna made the number 0.652. Whose answer do you agree with? Explain why.

Good response
I agree with Alex because with the digits 2, 5, and 6, I know 2 has the smallest value. So, I put the 2 in the tenths place because the tenths place is the biggest place value. You need to put the smallest number in the biggest place value if you are trying to make the smallest number. Then I put the 5 in the hundredths place because that is the second smallest number, and the hundredths place is the second biggest place value. All that was left was the 6 to put in the thousandths place. I knew Dayna was wrong because she had the largest number in the largest place value, which makes a large number, not a small one.

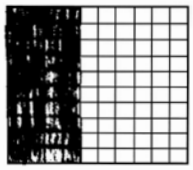
Underdeveloped response
Alex is right because 0.256 is smaller than 0.652.

FIGURE 4

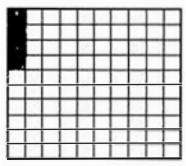
During a minilesson, students added information (in red) to responses.

When asked to shade 4 tenths, Corey shaded 4 rows. David shaded 4 squares. Do you agree with Corey or David? Explain why.

Corey



David



I agree with Corey because one row equals one tenth. I know that because one row has ten squares. and Corey shaded 4 rows which is 4 tenths. David shaded 4 little squares which is 4 hundredths not 4 tenths. because he shaded 4 squares out of a hundred squares. Because of this Corey is right and David is wrong.

verbal ideas into written explanations. Therefore, we followed a consistent approach for all discussions.

First, students read the question aloud, and we asked them what they thought the question was asking. For example, while working on the Becky-and-Lisa prompt (see first sample in **fig. 1**), students realized that each girl tried to add the same fractions but got different answers; our students' task was to explain why they agreed or disagreed with the girls' answers. Next, our students solved the problem themselves and then compared their answers with the two arguments presented, such as Becky's and Lisa's positions. Inevitably, because the prompts focused on a common misconception, some students agreed with one argument (e.g., "Becky added one and one right, and two and three right"); others agreed with the second stance (e.g., "I know that you can't add the bottom numbers, so Lisa must be right"); and still others failed to work out the solution fully (e.g., "I think that Lisa is right because the answer is almost one whole, but I am not sure how she got five-sixths").

We then had students compare their own solution strategies with those in the prompt and debate why they agreed with one, both, or none of the given positions. Inevitably, disagreements with any other answer, either those presented in the prompt or offered by a peer, fostered a more in-depth discussion that allowed students to unpack the mathematics. Over time, the conversations became primarily student directed, so they progressed in a variety of ways. For instance, some students who were working on the Becky-and-Lisa prompt used pictures to show how 1/2 and 1/3 are equivalent, respectively, to 3/6 and 2/6, whereas others represented this algebraically. Last, each individual student then crafted his or her own response, building off the ideas discussed with the group.

Refine the writing

Eventually, student responses reflected the qualities we were seeking. Students like Danielle (see **fig. 2**) started to include a great deal of detail; address both sides of the argument; provide clear reasoning; and fully explain what they did, why they did so, and how that related to their answers. Nevertheless, getting

students to write in this fashion was a long, ongoing undertaking.

To start the journaling process, we provided students with sample responses to show them what we were seeking in a fully developed answer and what an insufficient response would look like (see **fig. 3**). The underdeveloped samples demonstrated a lack of detail; the model responses showcased support of and explanation of mathematical ideas. By using a minilesson in which students worked as a group to revise a previously completed entry (see **fig. 4**), we were able to help students generate a list of aspects that should be included in a response to make it a high-quality one (see **the sidebar** on **p. 552**). Students came to realize what information their own responses were lacking and what information they were assuming the reader already knew.

Move forward in the process

After helping students understand the aspects of a quality response, we continuously nudged them toward discussion and writing that included reasoning, details, and fully supported claims. Using the journals as a formative assessment, we were able to tailor our instruction. Initially, most students' responses were basic (see **fig. 5**) but grew in length and depth over time, as we addressed the quality of students' writing. At first, students did not consider that the person they were writing to would not know what they were thinking. Our feedback during minilessons helped students reach the conclusion that their writing had to explain in a way that would convey their ideas to someone with no mathematical background. Once they grasped this concept, we saw improvements in the use of details and explanations in their responses.

As time progressed, students also began to explain their work and use pictures to support their reasoning. For example, Dina started by writing one sentence and did not supply ample reasoning (see **fig. 6**). Later on, she added ideas by using a picture to help explain her thought process. Although Dina's response did not fully address how she knew to circle four of the six roses, the use of a labeled diagram was a great improvement for this student. We continued to fine-tune all students' explanations over time and, for instance, would encourage Dina to explain how she came up with her answer.

FIGURE 5

At the beginning, student responses were quite basic.

When asked to shade 4 tenths, Corey shaded 4 rows. David shaded 4 squares. Do you agree with Corey or David? Explain why.

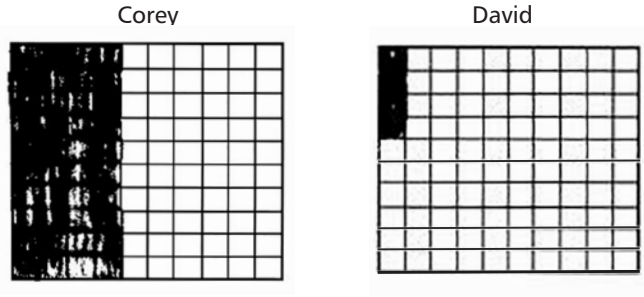


FIGURE 6

Dina showed improvement in the quality of her responses over time.

"Evan bought 6 roses for his mother, 2/3 of them red; how many roses were red?"

Jada's answer was 9. Luke answered 4. Do you agree with Jada or Luke? Explain why.

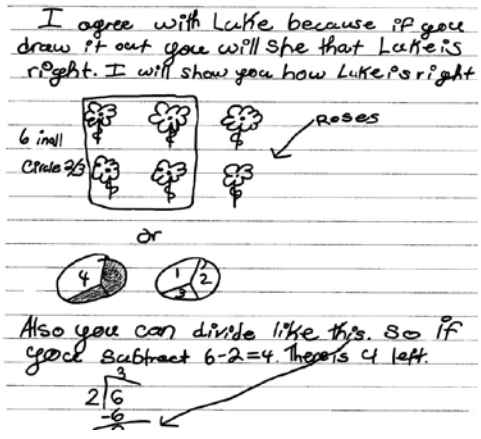
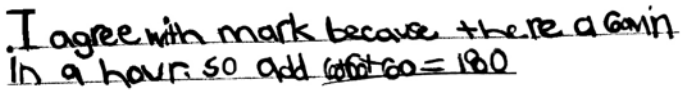


FIGURE 7

After teachers worked with Nick to verbally communicate who he agreed and disagreed with and why, the quality of his written responses noticeably improved.

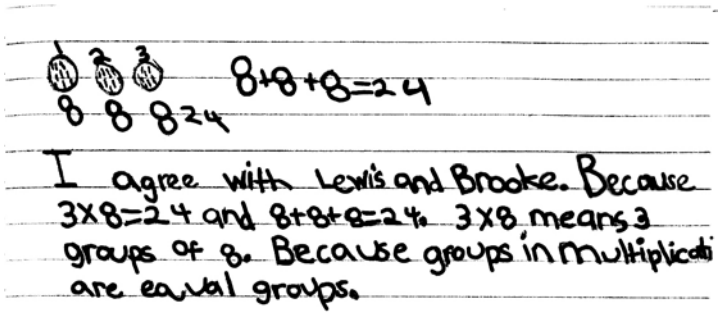
Response date 1/20/2013

Mark thinks there are 180 minutes in 3 hours. Tim thinks there are 120 minutes in 3 hours. Do you agree with Mark or Tim?



Response dated 3/20/2013

Lewis says that 3 × 8 stands for three groups of eight. Brooke says that this represents 8 + 8 + 8. Do you agree with Lewis or Brooke? Explain why.



Responses to math debate journal prompts

Below are some characteristics of a quality response to teacher prompts. (This is a student-generated list.)

- State who you agree or disagree with.
- Address *both* sides:
  - Discuss student A's thinking.
  - Discuss student B's thinking.
- Be clear.
- Use details.
- Explain fully.
- Use pictures to support your thinking.
- Make sure you are answering the question!

In journaling tasks, pairing writing and mathematical argumentation results in significant improvement of proficiency in both skills.

Differentiate the writing instruction

By formatively assessing the journals throughout the process, we were able to personalize instruction for the needs of our students. For instance, some students needed additional help seeing how to transition verbal ideas onto paper. Nick, for example, would write only a single sentence in his initial entries (see **fig. 7**). When working one-on-one, we had him verbally communicate who he agreed or disagreed with and why. We then recorded his ideas into notes for him to use during writing time. This way Nick could see how his ideas transferred into writing, and he eventually began to write more than one sentence, which showed great growth for him.

To help students move forward with the actual writing, we differentiated in additional ways. For instance, we gave students their own individual workspace around the room to help them focus. We also allowed students to use the free space in their journals as a place to collect their thoughts and take notes on the discussion. Some students also chose to paste the typed prompt into their journal and take notes on it, and this eventually served as their writing guide.

Hold high expectations

Overall, students enjoyed the challenge of writing in mathematics, and their skills vastly

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improved over time. They became better at communicating their reasoning and explaining their thinking. We held high expectations for students, and they stepped up to this challenge. The development in their writing conveyed deeper understanding of mathematical concepts than what they had previously realized or communicated. As an added benefit, students who used the debate journals will likely be more prepared for the upcoming challenges presented in the Common Core State Standard assessments and beyond. The tips we offer (see the **sidebar** on **p. 554**) highlight the components of our approach that teachers can implement with their own students.

Common Core Connections

SMP 3
SMP 6

⇢ reflect and discuss

Moving Math in the "Write" Direction

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions related to "Math Debate Journals: The 'Write' Direction," by Shannon E. Bostiga, Michelle L. Cantin, Cristina V. Fontana, and Tutita M. Casa, are suggested prompts to aid you in reflecting on the article and on how the authors' ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- How is the math debate journal similar to and different from other approaches to journaling in mathematics class?
- The authors of this article encourage their students to argue not only why a particular answer is correct but also why another answer is incorrect. How does this practice relate to the Common Core's Standards for Mathematical Practice?
- How do debate journals help address the third mathematical practice, which in part calls for students to critique the reasoning of others?
- The authors of this article discuss how they view discussion of the debate journal prompts as being an integral part of the learning experience throughout the journal writing process. In what ways can discussion enhance the math debate journal experience before, during, and after students write?

Tell us how you used Reflect and Discuss as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. When submitting letters to *Teaching Children Mathematics* (**tcm@nctm.org**), please include Readers Exchange in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgment. Letters are also edited for style and content.

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Teacher tips for implementing math debate journals

- Create journal prompts that require students to consider concepts in depth.
- Start the process slowly with plenty of modeling.
- Begin with a minilesson and revisit the following when needed for support:
  - Student understanding of written feedback
  - Elements of a good response
  - How to productively discuss
- Support student discussion of mathematical concepts prior to writing.
- Remember that this is a long-term process.
- Allow time for peer review and feedback.
- Allow students to rewrite and edit previous entries.




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